SOME WAYS OF MEASURING THERMOPHYSICAL PROPERTIES BY TEMPERATURE WAVE METHODS

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The possibility is considered of using two-dimensional temperature waves for determining the thermal diffusivity and thermal conductivity of solid and free flowing bodies from a single experiment without introducing heat sinks into the sample. Conditions are found for the reliable achievement of the boundary conditions.

The many advantages of methods of determining thermophysical properties based on the laws of a regular thermal regime of the third kind (stabilized propagation of temperature waves) have been discussed in detail [1, 2]. Because of these advantages temperature wave methods are now widely used. Unfortunately, the majority of these methods require the introduction of thermal sinks into the sample, and in many cases this may be undesirable or even completely unacceptable. It is true that this factor is eliminated in the method developed theoretically for high temperatures (above 1000° K) [4] and realized using electronic heating in vacuum and photometric recording of the temperature oscillations [5, 6]. However, this method [4] does not make it possible to determine the quantities a_1, λ_1, and $c_1\gamma_1$ from a single experiment and is unsuitable for heat-insulating materials.

A method has been put forward [2] for determining a_1 and λ_1 which does not require the introduction of heat sinks into a sample, but involves double measurements at different frequencies. Repeated measurements are obviously undesirable. Proceeding from the theoretical considerations in [2, 3], it can be shown that it is sufficient to carry out the measurements at one frequency if, in processing the measurements, use is made of both the amplitude and phase relationships of the recorded temperature oscillations.

Let some infinite plate, whose thermophysical properties are to be determined be in thermal contact with a second infinite plate (standard) having known values of a_2 and λ_2. Temperature oscillations are recorded on the "free" surface of the first plate and in the contact plane of the plates. Under experimental conditions the "free" surface of the first place is usually in contact with the source of temperature oscillations by a thin layer whose thermophysical properties are not reflected in the standard working formulae. If the thickness of the standard plate is sufficiently large it can be considered a semi-infinite body. Using, in this case, Laplace transforms to solve for the thermal conductivity and converting from the transfer temperature function in the contact plane to the corresponding amplitude-phase frequency characteristic, we obtain for the latter the expression

$$\Phi = (1+h)/(1+h)\,\mathrm{ch}\left(\varkappa\sqrt{\frac{i}{2}}\right) + (1-h)\,\mathrm{sh}\left(\varkappa\sqrt{\frac{i}{2}}\right). \qquad (1)$$

Writing this in the form

$$\Phi = M\exp(i\,\delta), \qquad (2)$$

it is possible [3] to obtain, from the expressions for M and δ, the functions $h = h(\varkappa, M)$ and $h = h(\varkappa, \delta)$ in the following form:

$$h = M^2\cos\varkappa - 1 + \Big[(M^2\cos\varkappa - 1)^2 - [1 - M^2\exp(-\varkappa)](1 - M^2\exp\varkappa)\Big]^{1/2}\left(1 - M^2\exp(-\varkappa)\right)^{-1} \qquad (3)$$

$$h = \frac{\mathrm{tg}(\varkappa/2) + \mathrm{tg}\,\delta}{\mathrm{tg}(\varkappa/2) - \mathrm{tg}\,\delta}\exp\varkappa. \qquad (4)$$

The quantities M and δ occurring in these equations are found directly from experiments. If expressions (3) and (4) are considered as a system of equations relating $\varkappa$ and h, then having solved it the unknown quantities a_1, λ_1, and $c_1\gamma_1$, can be evaluated in the following form:

$$a_1 = \frac{2\omega R^2}{\varkappa^2}, \quad \lambda_1 = \lambda_2\sqrt{\frac{a_1}{a_2}}\,\frac{1+h}{1-h}, \quad c_1\gamma_1 = \frac{\lambda_1}{a_1}. \qquad (5)$$

The system of transcendental Eqs. (3) and (4) allows only a numerical solution, which can be obtained using a nomogram (Fig. 1). In the rectangular coordinates h, $\varkappa$ two families of curves are constructed: $h = h(\varkappa)$ for various values of the parameter M according to formula (3), and $h = h(\varkappa)$ for various values of the parameter δ according to formula (4). The crossing points of the two curves, corresponding to experimental values of M and δ, give the unknown values of the quantities h and $\varkappa$. In the working nomogram, the following limiting values are provided for: $\varkappa$, from 0.75 to 3.50; h, from −0.80 to 0.80; M, from 0.10 to 0.80; $-\delta$, from 0.30 to 1.72. The scale along the coordinate axes is 0.005 per 1 mm length, and the difference between the values of a parameter corresponding to two neighboring curves in each family is equal to 0.01. Figure 1 shows a large part of the nomogram on a reduced scale.

From formula (5) and Fig. 1 it is seen that the nomogram is most accurate in the region close to $h = 0$, $\varkappa = \pi$.

Let us find the conditions in which the standard, a plate under actual conditions, can be considered a half-space (a semi-infinite body). A standard plate of thickness appreciably greater than the minimum necessary is undesirable, since it increases either the heating time for the system sample-standard to reach

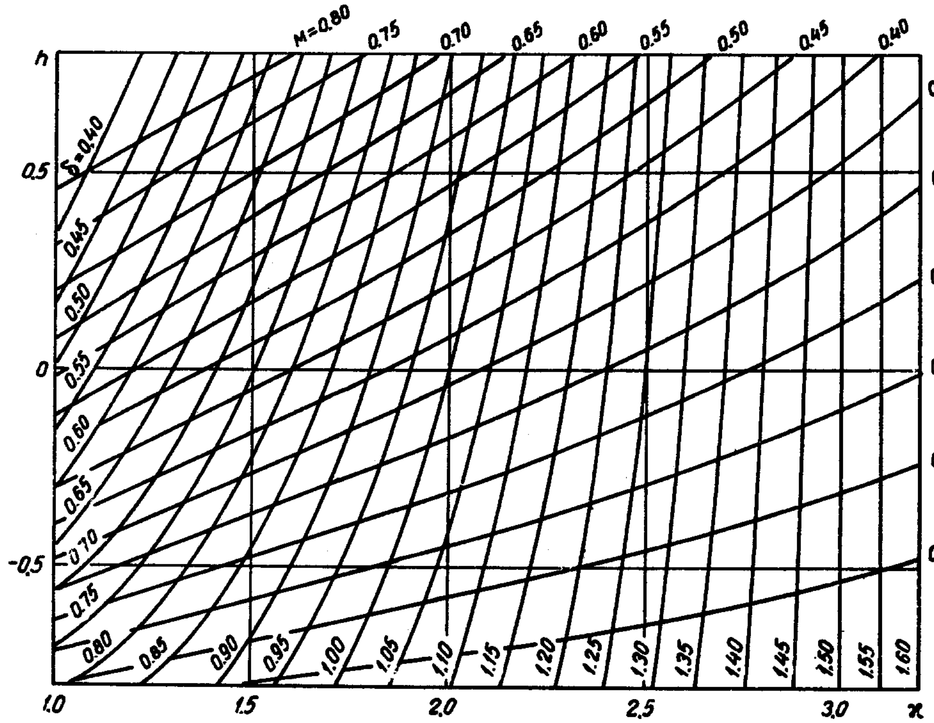

Fig. 1. Nomogram for determining the auxiliary quantities h and ϰ from values of M and δ.

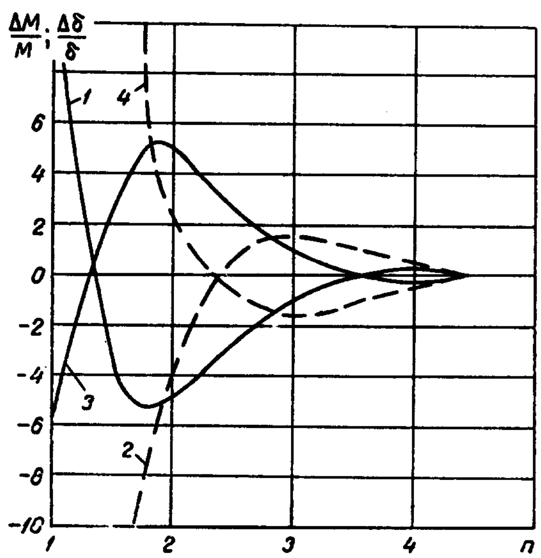

Fig. 2. The variation in the relative errors of M and (%), caused by the finite measurements of the standard body, with $n = \varkappa'/\varkappa$ at $h = 0$, $\varkappa = \sqrt{2}$: 1—$\Delta M_0/M$; 2—$\Delta\delta_0/\delta$; 3—$\Delta M_\infty/M$; 4—$\Delta\delta_\infty/\delta$.

a constant average temperature, or the internal drop in temperature, should the average temperature of the system change continuously during the measurement (cf., for example, [2]). Let us suppose that convective heat exchange occurs on the free surface of the standard plate of finite thickness. In the case of a standard plate of finite thickness, having obtained, using Laplace transforms, the transfer temperature function in the plane of contact of the plates, and converting to the corresponding amplitude-phase frequency characteristic ψ with respect to the harmonic temperature effect on the free surface of the first plate, we obtain

$$\Psi=\left\{(1+h)\left[\mathrm{Bi\,sh}\left(\varkappa'\sqrt{\frac{i}{2}}\right)+\varkappa'\sqrt{\frac{i}{2}}\times\right.\right.$$
$$\left.\left.\times\mathrm{ch}\left(\varkappa'\sqrt{\frac{i}{2}}\right)\right]\right\}\times\left(\mathrm{Bi}\left\{\mathrm{sh}\left[(\varkappa+\varkappa')\sqrt{\frac{i}{2}}\right]-\right.\right.$$
$$\left.-h\,\mathrm{sh}\left[(\varkappa-\varkappa')\sqrt{\frac{i}{2}}\right]\right\}+$$
$$+\varkappa'\sqrt{\frac{i}{2}}\left\{\mathrm{ch}\left[(\varkappa+\varkappa')\sqrt{\frac{i}{2}}\right]+\right.$$
$$\left.\left.+h\,\mathrm{ch}\left[(\varkappa-\varkappa')\sqrt{\frac{i}{2}}\right]\right\}\right)^{-1}. \qquad (6)$$

The least favorable cases are the limits Bi = 0 and Bi $\to\infty$. The amplitude-phase frequency characteristics ψ_0 and ψ_∞ corresponding to these cases will be equal to

$$\Psi_0=(1+h)\,\mathrm{ch}\left(\varkappa'\sqrt{\frac{i}{2}}\right)\Big/\mathrm{ch}\left[(\varkappa+\varkappa')\sqrt{\frac{i}{2}}\right]+$$
$$+h\,\mathrm{ch}\left[(\varkappa-\varkappa')\sqrt{\frac{i}{2}}\right], \qquad (7)$$

$$\Psi_\infty=(1+h)\,\mathrm{sh}\left(\varkappa'\sqrt{\frac{i}{2}}\right)\Big/\mathrm{sh}\left[(\varkappa+\varkappa')\sqrt{\frac{i}{2}}\right]-$$
$$-h\,\mathrm{sh}\left[(\varkappa-\varkappa')\sqrt{\frac{i}{2}}\right]. \qquad (8)$$

Let us consider the relationships

$$\frac{\Psi_0}{\Phi}=\frac{M+\Delta M_0}{M}\exp(i\,\Delta\delta_0),$$
$$\frac{\Psi_\infty}{\Phi}=\frac{M+\Delta M_\infty}{M}\exp(i\,\Delta\delta_\infty). \qquad (9)$$

Here the quantities ΔM_0 and ΔM_∞ are obviously the absolute errors in the ratios of amplitudes of the temperature oscillations, caused by the finite thickness of the standard plate at Bi = 0 and B $\to\infty$, respectively, and $\Delta\delta_0$ and $\Delta\delta_\infty$ are the analogous absolute errors of the phase differences.

Using formulae (1), (7), and (8) we obtain (9) in the following form:

$$\frac{\Psi_0}{\Phi}=$$
$$=\frac{\mathrm{ch}\,(n\varkappa\sqrt{i/2})\,[(1+h)\,\mathrm{ch}\,(\varkappa\sqrt{i/2})+(1-h)\,\mathrm{sh}\,(\varkappa\sqrt{i/2})]}{\mathrm{ch}\,[(n+1)\varkappa\sqrt{i/2}]+h\,\mathrm{ch}\,[(n-1)\varkappa\sqrt{i/2}]},$$

$$\frac{\Psi_\infty}{\Phi}=$$
$$=\frac{\mathrm{sh}\,(n\varkappa\sqrt{i/2})\,[(1+h)\,\mathrm{ch}\,(\varkappa\sqrt{i/2})+(1-h)\,\mathrm{sh}\,(\varkappa\sqrt{i/2})]}{\mathrm{sh}\,[(n+1)\varkappa\sqrt{i/2}]-h\,\mathrm{sh}\,[(n-1)\varkappa\sqrt{i/2}]}. \qquad (10)$$

The errors ΔM_0, ΔM_∞, $\Delta\delta_0$ and $\Delta\delta_\infty$ at h = 0 and $\varkappa=\sqrt{2}$ were evaluated from Eqs. (10). The variation of the relative errors $\Delta M_0/M$, $\Delta M_\infty/M$, $\Delta\delta_0/\delta$ and $\Delta\delta_\infty/\delta$ (in per cent) with the quantity n are shown in Fig. 2. In these calculations use was made of Table 7.3 in monograph [7], which demands little precision in the region n > 2.5. From Fig. 2 it is seen that for the chosen values of $\varkappa$ and h, both for Bi = 0 and Bi $\to\infty$, both errors do not exceed 1% if n > 4; for n > 4.5 the errors caused by the finite thickness of the standard plate are almost zero with the most precise measurements. In practice values of $\varkappa>\sqrt{2}$ are usually encountered. Calculations show that in this case values of n can be chosen which are even smaller. For example, for $\varkappa=2\sqrt{2}$, a value of n = 4 is sufficient for measurements of any precision. Calculations also show that by choosing values of n in conformity with these criteria the change in h in the interval [−1, 1] introduce almost no error into the values of M and δ determined experimentally.

NOTATION

R is the thickness of the first infinite plate (sample); a_1, λ_1, c_1 and γ_1 are the corresponding thermal diffusivity, thermal conductivity, specific heat capacity, and density of the material of this plate (determinable quantities); L is the thickness of the second infinite plate (standard); a_2 and λ_2 are the corresponding thermal diffusivity and thermal conductivity of the material of the second plate; M is the ratio of the amplitude of the temperature oscillations in the plane of contact of the plates to the corresponding amplitude on the free surface of the first plate at L $\to\infty$; δ is the phase difference of these oscillations ω is the heat exchange coefficient on the free surface of the second plate; $\mathrm{Bi}=\alpha L/\lambda_2$ is Biot's criterion; $h=(\lambda_1\sqrt{a_2}-\lambda_2\sqrt{a_1})/(\lambda_1\sqrt{a_2}+\lambda_2\sqrt{a_1})$; $\varkappa=\sqrt{2\omega/a_1}\,R$; $\varkappa'=\sqrt{2\omega/a_2}\,L$; $n=\varkappa'/\varkappa$.

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